In Nakamura, T., Orchiston, W., Sôma, M., and Strom, R. (eds.), 2011. *Mapping the Oriental Sky. Proceedings of the Seventh International Conference on Oriental Astronomy*. Tokyo, National Astronomical Observatory of Japan. Pp. 171-178.

ASTRONOMY AND MATHEMATICS OF YIXING

Yukio ÔHASHI (大橋由紀夫)
3-5-26, Hiroo, Shibuya-ku, Tokyo 150-0012, Japan.
E-mail: yukio-ohashi@dk.pdx.ne.jp

Abstract: Yixing was a Chinese Buddhist monk and astronomer in the Tang Dynasty of China. It is well known that he made an excellent calendar entitled *Dayan-li* in AD 727, which was once used also in Japan. In this paper, I would like to discuss his contribution in the field of astronomy and mathematics. I shall discuss a possible influence of Indian astronomy on Yixing in his new definition of *mieri* in the *Dayan-li*. And also, I would like to discuss his method of interpolation which was used in the *Dayan-li*. Yixing introduced some new methods of interpolation. The meaning of his interpolation is controversial, and I would like to present my own view regarding the origin of his interpolation method and compare it with another view presented by QU Anjing. Anyway, his method of interpolation can be understood as a natural development of Chinese interpolation during the Sui and Tang Dynasties.[1]

1 INTRODUCTION[2]

Yixing (一行)(AD 683–727)[3] was a Chinese Buddhist monk and astronomer in the Tang (唐) Dynasty (AD 618-907) of China. Yixing is his Buddhist name, and his secular name was Zhang Sui (張遂)(Zhang being his family name). He is sometimes called Seng Yixing (僧一行)(Monk Yixing) or Yixing chanshi (一行禪師)(Zen master Yixing).

Yixing received a call from Emperor Xuanzong (玄宗) in AD 717, and moved to Chang‟an (長安) (present-day Xi‟an (西安)), the then capital of China. After that, Yixing learnt Esoteric Buddhism from Indian monks Śubhakarasiṁha (whose Chinese name is Shan-wuwei (善無畏)) and Vajrabodhi (whose Chinese name is Jingang-zhi (金剛智)).

In AD 721, at the Emperor‟s request Yixing began a project to make a new calendar. He also made an armillary sphere with his colleague Liang Lingzan (梁令瓚) in around AD 724, and observed stars. From AD 724, Yixing conducted astronomical observations at several places throughout China with his colleague Nangong Yue (南宮說). In AD 725, Yixing made a water-driven celestial globe with Liang. After these preparations, Yixing started to compile a new calendar, and completed the draft of the new *Dayan-li* (*Dayan* Calendar, the word "*li*" stands for "calendar" or "ephemeris".) (大衍曆) in AD 727, but he died later that year. Zhang Shui (張說) and Chen Xuanjing (陳玄景) then edited Yixing‟s draft,[4] and the *Dayan* Calendar was officially used from AD 729.

2 YIXING'S ASTRONOMICAL CONTRIBUTION

2.1 Introduction

In the Sui (隋)(AD 581-618) and Tang (唐)(AD 618-907) Dynasties, several good calendars were made. A rough history of calendars before Yixing in this period is as follows. The *Huangji* Calendar (皇極曆)(AD 600) of Liu Zhuo (劉焯)(AD 544–610) was not officially used, but was an excellent calendar where the inequalities corresponding to the equation of centre of the Sun and of the Moon and the precession of the equinoxes were considered, and also the second order interpolation was used for the first time in China. The *Linde* Calendar (麟德曆)(665 AD) of Li Chunfeng (李淳風)(AD 602–670) is also a famous calendar. Li is also famous for his armillary sphere. The *Linde* Calendar was also used in Korea and Japan.

Then Yixing composed his *Dayan* Calendar, which was one of the best calendars of the Tang Dynasty. After Yixing, the *Xuanming* Calendar (宣明曆)(AD 822) of Xu Ang (徐昂) is also famous, and the method of the prediction of eclipses was improved. And also, the *Chongxuan* Calendar (崇玄曆)(AD 892) of Bian Gang (邊岡) also contains several devices. The *Dayan* Calendar and the *Xuanming* Calendar were used in Japan while the *Xuanming* Calendar was also used in Korea.

The Tang Dynasty is also a period when Indian astronomy was introduced to China. According to my research, some information about Indian astronomy might have reached China in the Later (Eastern) Han (東漢) Dynasty. (See Ôhashi (1994a and 1999a) and the following section (2.4).) After that, a Buddhist text containing the knowledge of Indian astrology and astronomy, the *Śārdūlakarṇa-avadāna*, was translated into Chinese in the „Three Kingdoms‟ (三国) Period in the third century AD. In the Tang Dynasty, a detailed work on Indian mathematical astronomy, the *Jiuzhi* Calendar (九執曆)(AD 718), was composed in Chinese by an Indian astronomer named Qutan Xida (瞿曇悉達)(Chinese transliteration of Gotama-siddha in Sanskrit), and was included in his ((*Da-*)*Tang-*) *Kaiyuan-zhanjing* (((大)唐)開元占經). Qutan had been resident in China since the time of his grandfather.

Yixing also had some knowledge of Indian astronomy, but his *Dayan* Calendar followed the traditional Chinese style. This fact should not be forgotten.

In the eighth century AD, a Chinese version of Indian astrology, the *Xiuyao-jing* (宿曜經), was composed in Chinese by an Indian monk, Bukong (不空)(whose Sanskrit name was Amoghavajra)(705–774 AD). Amoghavajra was a disciple of Vajrabodhi, with whom Yixing also studied.

2.2 Observational Astronomy

Yixing and Liang Lingzan (梁令瓚) made an armillary sphere which was called Huangdao-youyi (黃道游儀)(Instrument with a Movable Ecliptic Circle) in around 724 AD. As the name might suggest, on this instrument the ecliptic circle could be moved in accordance with the precession of the equinoxes. It also had a movable circle for lunar orbit. With this instrument, Yixing observed stars, particularly in the 28 lunar mansions, and, comparing them with previous observations, measured changes in their polar distances and right ascensions. These change were due to the precession of the equinoxes.

Yixing and Liang also made a water-driven celestial globe in 725 AD. Besides the celestial globe, it had two wooden figures which automatically struck a drum and a gong.

From 724 to 725 AD, Yixing and Nangong Yue (南宮說) conducted astronomical observations at thirteen different locations between about 51° N to about 18° N. They observed the altitude of the North Celestial Pole, the length of the gnomon-shadow at the solstices and the equinoxes, and the length of day-time and night-time during the solstices.

2.3 Theoretical Astronomy

Yixing‟s *Dayan* Calendar was one of the best calendars in China, and had several significant features.

In China, the inequality corresponding to the equation of centre of the Sun was discovered by Zhang Zixin (張子信) in the sixth century AD, independently of Hipparchus‟ discovery in Ancient Greece. According to this inequality, Yixing gave the values for twenty-four points of time in a year, which were divided according to the Sun‟s angular movement. Here, Yixing used second order interpolation with unequal steps of argument for the first time in China.

For the inequality corresponding to the equation of the centre of the Moon, which was discovered in the first century AD in the Later Han Dynasty in China, Yixing used second order interpolation with equal steps of argument invented by Liu Zhuo (劉焯)(542–608 AD) in the Sui Dynasty.

An attempt to predict lunar eclipses was started by the *Santong* Calendar (三統曆) of Liu Xin (劉歆)(d. 23 AD) at the end of the Former Han Dynasty, and the basis of the standard system of the prediction of solar and lunar eclipses was established by the *Jingchu* Calendar (景初曆)(237 AD) of Yang Wei (楊偉) in the Three Kingdoms Period. For the prediction of solar eclipses, Yixing considered the lunar parallax at different places. Although his method was not perfect, it was a big step forward. The method to predict eclipses was later developed further in the *Xuanming* Calendar (宣明曆)(822 AD) by Xu Ang (徐昂).

Yixing also improved the calculation of the position of the five planets, and used a type of interpolation in which the third difference is used, although it was not the interpolation of the third order.

Another one of Yixing‟s contributions was a device to calculate the length of the gnomon-shadow and the length of day-time and night-time in different seasons and at different places. For this purpose, he made a table of the gnomon-shadow for every „Chinese degree‟,[5] from 0 to 81, of the Sun‟s zenith distance. This table is the earliest tangent table in the world (see Qu, 1997).

For the transformation of spherical coordinates, the graphical method on the celestial globe was used from the Later Han Dynasty. An arithmetical method was started in the *Huanji* Calendar (皇極曆) (600 AD) by Liu Zhuo (劉焯), and Yixing also used an arithmetical method. In this method, the difference between right ascension and polar longitude (longitude of the requisite hour circle on the ecliptic) was assumed to be a linear function in a quadrant, and the difference was given in a table.

Yixing‟s *Dayan* Calendar was introduced to Japan, and was officially used there from 746 to 857 AD.

2.4 Yixing and Indian Astronomy

Yixing had some knowledge of Indian astronomy. For example, he mentioned the Indian zodiac in his *Dayan* Calendar, but he made his *Dayan* Calendar in Chinese traditional way. However, there is one area where I suspect there is Indian influence in his Calendar, and this is the change of the meaning of *mieri* (滅日) (for my interpretation of *mori* and *mieri* see Ôhashi, 1999b; 2000).

Let us first see the meaning of this word before Yixing used it. The Later Han *Sifen* Calendar (後漢四分曆)(85 AD) had special days called *mori* (沒日) and *mieri*, which did not exist in the Former Han Dynasty. If one year was divided into 360 parts and one day was included within a part, the day was called *mori*. If the end of a day coincided with the boundaries of the parts, the day was called *mieri* These days were of no use in traditional Chinese calendars, but were similar to certain concepts in traditional Indian calendars, such as the method of intercalation in the *Artha-śāstra*.

Now, Yixing changed the meaning of *mieri*, explaining the method of calculating it as follows (my English translation):

> If the *xiaoyu* (time in terms of 1/3040 day) of the mean new Moon is less than *shuoxufen* (= 1427), subtract the *xiaoyu* from the *tongfa* (= 3040), and multiply the result by 30, and subtract the result from *miefa* (= 91200). Divide the result by *shuoxufen*. The result is the number of days. Count the days from the mean new Moon day and take the day after the resultant day. It is the *mieri*. (*Xin-Tangshu, Lizhi*, Zhonghua-shuju, 1976, Volume 7: 2219).

The meaning of this *mieri* is as follows: let a synodic month be divided into 30 parts. Then, sometimes a part is included within a day. This kind of day is the *mieri* defined by Yixing.

This *mieri* of Yixing is similar to the „omitted *tithi*" in Indian calendars. In *Vedāṅga* astronomy, a *tithi* was a 1/30 part of a synodic month, where the equation of centre was not known. In Hindu Classical Astronomy, a *tithi* is a period of time during which the longitudinal difference of the Sun and the Moon changes by 12°. If a *tithi* is included within a day, the *tithi* is called „omitted *tithi*". In Hindu traditional calendars, the name of a civil day is determined by the number of *tithi* at the beginning (sunrise) of the day. Therefore, the number of an „omitted *tithi*", which does not include any sunrise, actually disappears from the calendar.

The significance of Yixing‟s definition is that when the sum of the *mori* (a day which is included within a segment of 1/360 of a tropical year) and *mieri* grows up to 30, one intercalary month is produced. This way of thinking is similar to a certain description in the Indian classics, such as the *Artha-śāstra*. A similar description is also found in a Chinese version of the Buddhist text *Lishi-apitan-lun* (立世阿毘曇論), which was translated by Zhendi (真諦) in the middle of the sixth century AD.

I suspect that Yixing knew of this Indian method and changed the meaning of *mieri* in order to make it meaningful in an Indian calendrical context.

3 YIXING'S MATHEMATICAL CONTRIBUTION: THE DEVELOPMENT OF INTERPOLATION[6]

3.1 Introduction

Interpolation is a method to estimate a value of a function $f(x)$ from its discrete values (for example, $f(0)$, $f(1)$, $f(2)$, etc.). For this purpose, differences (for example, „first order differences" such as $\Delta_1 = f(1) - f(0)$, $\Delta_2 = f(2) - f(1)$ etc., and „second order differences" such as $\Delta^2{}_1 = \Delta_2 - \Delta_1$ etc.) are used.

The second order interpolation established at the time of the Sui Dynasty originated in the first order interpolation applied to the first differences. At the time of the Tang Dynasty, Yixing started to use the third difference, but his method was not third order interpolation. Yixing applied the first order interpolation to the second differences (in other words, applied the second order interpolation to the first differences), and the accuracy of the result was not so different from the previous second order interpolation. Yixing‟s method was a natural development of the method used during the Sui period.

More than ten years ago, I published two papers regarding Chinese interpolation (Ôhashi, 1994b; 1995). At that time, some young Chinese researchers were also studying Chinese mathematical astronomy including interpolation (e.g. see Qu, Ji and Wang, 1994). Among them, Qu Anjing (曲安京) has recently published a detailed monograph on Chinese mathematical astronomy (Qu, 2005). Qu‟s interpretation of Yixing‟s interpolation using the third difference is quite different from my own interpretation (e.g. see Ôhashi, 2009b), and I would like to discuss this matter in this paper.

3.2 Chinese Interpolation before Yixing

The earliest record of second order interpolation in China is found in the *Huangji* Calendar (皇極暦)(AD 600) in the Sui Dynasty, and a similar method was also used in the *Linde* Calendar (麟徳暦)(officially used from AD 665) in the Tang Dynasty. Li Shanlan (李善蘭) explained this interpolation using a geometrical model in his *Linde-shu-jie* (麟徳術解)(AD 1848). However, I think that this interpolation originated from an arithmetical method (see Ôhashi, 1994b)).

In the actual calculation in this interpolation, Δ in the table given in the calendar is an arithmetical progression (Δ^2 is constant). Therefore the second order interpolation can be considered to be perfect.

When Δ_1, Δ_2 etc. are given, the following values are obtained:

$$\Delta_{1.5} = \frac{\Delta_1 + \Delta_2}{2} \qquad \Delta_{2.5} = \frac{\Delta_2 + \Delta_3}{2}$$

As Δ is an arithmetical progression, this calculation is exact. From these values, every day‟s (or every degree‟s) value of Δ can be calculated, and their sum total for a certain period can be considered to be δ (see below.). (For example, the value of one day‟s Δ for the $(\alpha + 1)$th day in the period of n days (from $\Delta_{1.5}$ to $\Delta_{2.5}$) is considered to be: $\Delta_{1.5}/n + \alpha(\Delta_{2.5}/n - \Delta_{1.5}/n)/n$ (and here $(\Delta_{2.5}/n - \Delta_{1.5}/n) = (\Delta_3/n - \Delta_2/n)$ etc.). Actually, this method has a half-day‟s error, and the multiplier of the second term should be $\{\alpha + (1/2)\}$ instead of α. This error may be called "half-day problem". Later Yixing corrected this error.)

In order to calculate $f(n + \alpha)$ from $f(n)$, $f(2n)$ etc., the value of δ (let $f(n + \alpha) = f(n) + \delta$) is obtained. The value of δ can be obtained by an application of linear interpolation of $\Delta_{1.5}$, $\Delta_{2.5}$ etc. as above, and is applied to $f(n)$. This is the method of the second order interpolation of f.

Yixing started a new method of interpolation where Δ is not an arithmetical progression. In this case, $\Delta_{1.5}$, $\Delta_{2.5}$ etc. cannot be obtained by linear interpolation exactly (therefore Yixing used the third difference). After obtaining $\Delta_{1.5}$, $\Delta_{2.5}$ etc., Yixing used a method which is similar to the previous method to calculate f.

3.3 My (Ôhashi's) Interpretation and QU Anjing's Interpretation

Yixing used interpolation using the third difference in the calculation of eclipses and the calculation of five planets in his *Dayan* Calendar.

Among them, the beginning of the interpolation used for the calculation of lunar polar longitude in the calculation of eclipses is as follows (after *Xin-Tangshu*, *Lizhi*, Zhonghua-shuju, 1976, Volume 7: 2247; my tentative English translation):

(1) The difference between *qi-yao-jiajianlü* (Δ_2) and the *hou-yao-jiajianlü* (Δ_3) is *qian-cha* (Q).
(2) The difference between the *hou-yao-jiajianlü* (Δ_3) and the *ci-hou-yao-jiajianlü* (Δ_4) is *hou-cha* (H).
(3) The difference between the two differences (Q and H) is *zhong-cha* (Z).
(4) Put the sum of the *qi-yao-jiajianlü* (Δ_2) and the *hou-yao-jiajianlü* (Δ_3), a half of the *zhong-cha* (Z) is added to it, then halve it, and divide it by 15. The result is the *yao-molü* (M, the last value of the segment B_1B_2). It is the same as the *hou-yao-chulü* (the first value of the next segment B_2B_3).

Note that one segment (such as B_1B_2 in Figure 1), *yao*, consists of n (= 15) *du* (Chinese degrees). After determining the first and the last value of the segment, the required portion of the first difference (Δ) is calculated by the application of the first order interpolation, and the required *f(x)* is determined. So, the accuracy of *f(x)* is similar to the previous second order interpolation.

Let the *qi-yao-jiajianlü*, *hou-yao-jiajianlü*, and *ci-hou-yao-jiajianlü* be Δ_2, Δ_3, and Δ_4 (E_1, E_2, and E_3 in Figure 1) respectively. And also, let the *qian-cha*, *hou-cha*, and *zhong-cha* be Q, H, and Z respectively. Let the result, *yao-molü*, be M. Here, M corresponds to $\Delta_{2.5}$.

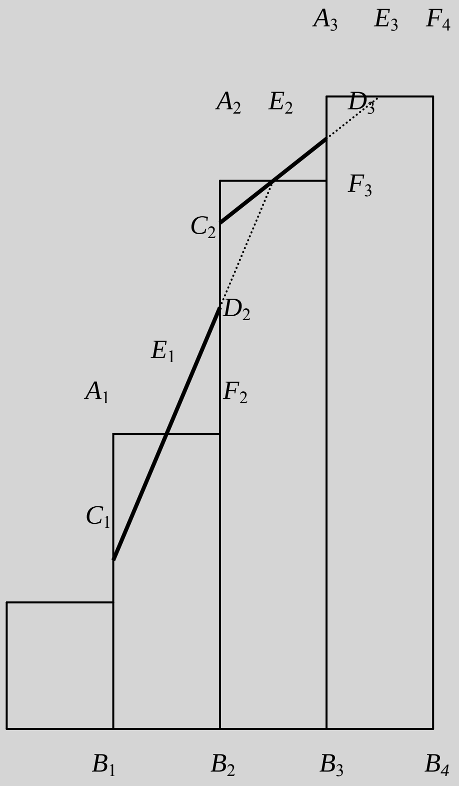

Figure 1: After Qu Anjing, 2005:250.

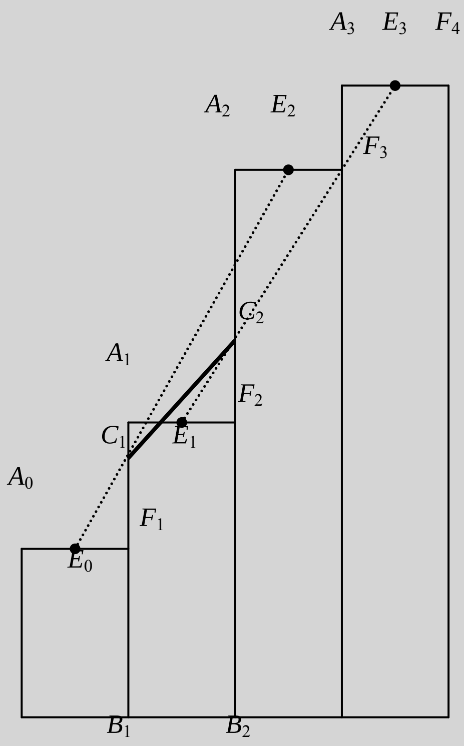

Figure 2: After Qu Anjing, 2005: 251.

My (Ôhashi's) interpretation of the above method is as follows.

(1) $Q = |\Delta^2_2|$, $(\Delta^2_2 = \Delta_3 - \Delta_2)$

(2) $H = |\Delta^2_3|$, $(\Delta^2_3 = \Delta_4 - \Delta_3)$

(3) $Z = |\Delta^3_2|$, $(\Delta^3_2 = H - Q = |\Delta^2_3| - |\Delta^2_2|$, which is always negative in this calendar.

(4) $$M = \frac{|\Delta_2| + |\Delta_3| + \frac{|\Delta^3_2|}{2}}{2n} = \frac{\Delta_2 + \Delta_3 - \frac{\Delta^3_2}{2}}{2n}$$

Here, Δ_2 and Δ_3 are originally positive in the calendar.

Qu Anjing's interpretation is as follows:

(1) $Q = \Delta^2{}_2 = \Delta_3 - \Delta_2$

(2) $H = \Delta^2{}_3 = \Delta_4 - \Delta_3$

(3) $Z = \Delta^3{}_2 = \Delta^2{}_3 - \Delta^2{}_2$

(4) $M (= B_2C_2) = \dfrac{\Delta_2 + \Delta_3 + \dfrac{\Delta_2^3}{2}}{2n} = \dfrac{3\Delta_2 + \Delta_4}{4n}$

In the method in the Sui Dynasty, first differences were arithmetical progression, but were not in Yixing"s interpolation (but second differences were arithmetical progression in Yixing"s interpolation). Qu Anjing (2005: 250) pointed out that there appears to be a discontinuity (the difference between D_2 and C_2 in Figure 1) if the old method is applied to Yixing"s progression, and that Yixing started his new method (see Figure 2).

3.4 Yixing's Interpolation According to my (Ôhashi's) Interpretation

Yixing"s method of using Δ^3 is first order interpolation applied to Δ^2. In the original text, Yixing"s definition of Δ^3 is the "… difference between the two differences." There is no sign in the „difference"; it should be an absolute value. Here, Δ is not an arithmetical progression, but Δ^2 *is* an arithmetical progression. Therefore, Yixing applied first order interpolation to Δ^2. (This is second order interpolation of Δ, but is not third order interpolation of *f*.) In order to determine *molü* (the last value of Δ of the segment B_1B_2), Yixing applied second order interpolation to Δ. If so, according to the modern interpolation, the *molü* should be $\dfrac{\Delta_2 + \Delta_3 - \dfrac{\Delta_2^3}{4}}{2n}$. However, Yixing used $\dfrac{\Delta_2 + \Delta_3 - \dfrac{\Delta_2^3}{2}}{2n}$. This may be a mistake by Yixing. I shall use Figure 2 to explain my viewpoint.

Qu Anjing"s interpretation is practically a first order interpolation of Δ_2 and Δ_4. In Figure 2, the points E_1 and E_3 represent Δ_2 and Δ_4 respectively. (Here, to use or not to use the multiplier $1/n$ is only a problem of the conversion of units, and is not so important. If one thinks that E_1 should be Δ_2/n, one may use Δ_2/n etc. in the calculation. The result is the same.) My interpretation is that Yixing"s method is second order interpolation applied to Δ_2, Δ_3 and Δ_4. In other words, it is second order interpolation applied to E_1, E_2 and E_3.

The modern (Newton"s) second order interpolation is:

$$f(1) - f(0) \equiv \Lambda_1, f(2) - f(1) \equiv \Lambda_2, \Lambda_2 - \Lambda_1 \equiv \Lambda_1^2,$$

$$f(x) = f(0) + x\Lambda_1 + \frac{x(x-1)}{2!}\Lambda_1^2$$

and in order to distinguish Newton"s and Yixing"s methods I used Λ in Newton"s formula instead of Δ.

Chinese interpolation was also like this. Yixing applied this kind of second order interpolation to Δ_2, Δ_3 and Δ_4. Now, let us consider as follows (using the points E_1, E_2 and E_3 in Figure 2 to explain the method):

$$f(0) = \Delta_2 (= E_1), f(1) = \Delta_3 (= E_2), f(2) = \Delta_4 (= E_3)$$

$$\Lambda_1 = \Delta_2^2, \Lambda_2 = \Delta_3^2$$

$$\Lambda_1^2 = \Delta_2^3$$

Here, the segmental distance between E_1, E_2 and E_3 is 1. So the segmental distance between E_1 and F_2 is 1/2. The *molü* (the last value of Δ of the segment B_1B_2) is $\dfrac{1}{n} f(\tfrac{1}{2})$.

Now, using the modern formula to calculate $f(\tfrac{1}{2})$:

$$f(\tfrac{1}{2}) = f(0) + \tfrac{1}{2}\Lambda_1 + \frac{\tfrac{1}{2}(\tfrac{1}{2}-1)}{2}\Lambda_1^2 = \Delta_2 + \tfrac{1}{2}\Delta_2^2 - \tfrac{1}{8}\Delta_2^3$$

$$= \tfrac{1}{2}\Delta_2 + (\tfrac{1}{2}\Delta_2 + \tfrac{1}{2}\Delta_2^2) - \tfrac{1}{8}\Delta_2^3$$

$$= \tfrac{1}{2}(\Delta_2 + \Delta_3 - \tfrac{1}{4}\Delta_2^3)$$

Therefore, the *molü* should be $\dfrac{\Delta_2 + \Delta_3 - \dfrac{\Delta_2^3}{4}}{2n}$. If one thinks that E_1 should be Δ_2/n, one may apply the second order interpolation to Δ_2/n, Δ_3/n and Δ_4/n, and calculate $f(\tfrac{n}{2})$. The result is the same.

Yixing"s result was $\dfrac{\Delta_2+\Delta_3-\dfrac{\Delta_2^3}{2}}{2n}$. This mistake is somewhat strange and might be related to the „half-day problem" (see section 3.2 above). Anyway, the reason for the appearance of Δ^3 is clear now. It was for the second order interpolation of Δ.

In order to apply earlier Chinese second order interpolation, Δ had to be an arithmetical progression. The tables given in the *Huangji*-calendar (Sui period) etc. were exactly like this. Here we can see the artificialness in the tables. The authors at that time had to make Δ a arithmetical progression in order to apply second order interpolation to *f*. In the table of Yixing (*Xin-Tangshu*, *Lizhi*, Zhonghua-shuju, 1976, Volume 7: 2246-2247), Δ^2 is an arithmetical progression, and Δ^3 is a constant with the value –8. Here, we see artificialness again. As Δ^2 is an arithmetical progression, this must be for applying second order interpolation to Δ. Here, we can see the evident purpose of Yixing.

3.5 Examples

Numerical examples actually in the calendar are as follows, using the part of the table listed below as an example:

f(0) = 0	*f*(1) = 187	*f*(2) = 358	*f*(3) = 505	*f*(4) = 620	*f*(5) = 695	*f*(6) = 722	---
Δ_1 = 187	Δ_2 = 171	Δ_3 = 147	Δ_4 = 115	Δ_5 = 75	Δ_6 = 27,	---	

Example 1 (Δ_2 = 171; Δ_3 = 147; Δ_4 =115)

$\Delta^2{}_2 = \Delta_3 - \Delta_2 = 147 - 171 = -24$

$\Delta^2{}_3 = \Delta_4 - \Delta_3 = 115 - 147 = -32$

$\Delta^3{}_2 = \Delta^2{}_3 - \Delta^2{}_2 = -32 - (-24) = -8$

My (Ôhashi"s) interpretation:

$$M=\frac{|\Delta_2|+|\Delta_3|+\dfrac{|\Delta_2^3|}{2}}{2n}=\frac{\Delta_2+\Delta_3-\dfrac{\Delta_2^3}{2}}{2n}=\frac{171+147-\dfrac{(-8)}{2}}{2n}=\frac{322}{2n}$$

Qu Anjing"s interpretation:

$$M=\frac{\Delta_2+\Delta_3+\dfrac{\Delta_2^3}{2}}{2n}=\frac{3\Delta_2+\Delta_4}{4n}=\frac{3\times171+115}{4n}=\frac{513+115}{4n}=\frac{628}{4n}=\frac{314}{2n}$$

Example 2 (Δ_8 = 75; Δ_9 = 115; Δ_{10} = 147)

$\Delta^2{}_8 = \Delta_9 - \Delta_8 = 115 - 75 = 40$

$\Delta^2{}_9 = \Delta_{10} - \Delta_9 = 147 - 115 = 32$

$\Delta^3{}_8 = \Delta^2{}_9 - \Delta^2{}_8 = 32 - 40 = -8$

My (Ôhashi"s) interpretation:

$$M=\frac{|\Delta_8|+|\Delta_9|+\dfrac{|\Delta_8^3|}{2}}{2n}=\frac{\Delta_8+\Delta_9-\dfrac{\Delta_8^3}{2}}{2n}=\frac{75+115-\dfrac{(-8)}{2}}{2n}=\frac{194}{2n}$$

Qu Anjing"s interpretation:

$$M=\frac{\Delta_8+\Delta_9+\dfrac{\Delta_8^3}{2}}{2n}=\frac{3\Delta_8+\Delta_{10}}{4n}=\frac{3\times75+147}{4n}=\frac{225+147}{4n}=\frac{372}{4n}=\frac{186}{2n}$$

4 CONCLUSION

The Chinese monk Yixing was a great figure in the field of astronomy as well as Buddhism. He contributed to the development of theoretical astronomy, observational astronomy and mathematics, and his *Dayan* Calendar resulted from his astronomical research, and was possibly influenced in a minor way by Indian astronomy. This is an interesting subject and, just like Yixing"s method of interpolation, should also be studied further.

5 NOTES

1. This paper is based on one of two poster papers presented at ICOA-7 (Tokyo, 2010). The other poster

paper titled "Mathematical Astronomy of Seki Takakazu and Shibukawa Harumi" was a revised and enlarged version of Ôhashi (2009a), and is not included in these *Proceedings* due to page limitations.
2. For the history of mathematical astronomy before Yixing, in English, see Ôhashi (2007; 2008). For an English language account of Chinese mathematical astronomy after Yixing, see the discussion about Guo Shoujing (郭守敬) (1231–1316) in Ôhashi (2009a). For the general history of Chinese astronomy, including Yixing‟s contribution, see Needham (1959), in English; Bo et al. (1981 and 2008-09), Chen Meidong (2003), and Chen Zungui (1980-89), in Chinese; and Yabuuti (1990), in Japanese.
3. The *Pinyin* system of Romanization (which is standard in Mainland China) is used in this paper. Note that Needham (1959) refers to Yixing as „I-Hsing‟. Classical bibliographies of Yixing are as follows. There is an epitaph to Yixing written by the Emperor Xuanzong (玄宗) (reign 712-756 AD), which is recorded in Kūkai (821). The official biography of Yixing is included in the section of biographies on technicians in the *Jiu-tang-shu* (Liu et al., 945), Volume 191. The classical biography of Yixing as a Buddhist monk is in Zanning (988), Volume 5. Some classical accounts of Yixing are collected in Ruan (1799). Modern biographies of Yixing as an astronomer are: Li (1964), Chen Meidong (1992; 1998) and Chen Jiujin (2008). For a study of Yixing as a Buddhist monk see Osabe (1963).
4. The system of Yixing‟s *Dayan* Calendar is recorded in the *Jiu-tang-shu* (Liu et al., 945), Volume 34, and in the *Xin-Tang-shu* (Ouyang et al., 1060), Volumes 27-28. The latter source also contains Yixing‟s theoretical exposition on the *Dayan* calendar and is an important treatise in Chinese astronomy. These astronomical treatises in the *Jiu-Tang-shu* and the *Xin-Tang-shu* are also included in the Zhonghua-shuju (1976). Yixing also composed several Buddhist works, but I will not discuss them here.
5. One „Chinese degree‟ is the angular distance on the celestial sphere through which the mean Sun moves in one day.
6. Another mathematical contribution by Yixing which is not discussed here has already been discussed by Qu (1997).

6 REFERENCES

Bo Shuren (薄树人) et al. (eds.), 1981. *Zhongguo tianwenxue-shi* (中国天文学史) (*History of Chinese Astronomy*). Beijing, Kexue-chubanshe (in Chinese).

Bo Shuren et al. (eds.), 2008-2009. *Zhongguo tianwenxue-shi daxi* (中国天文学史大系) (*Outline of the History of Chinese Astronomy*). Ten volumes. Beijing, Zhongguo-kexuejishu-chubanshe (in Chinese).

Chen Jiujin (陈久金), 2008. Yixing. In Chen Jiujin (ed.). *Zhongguo gudai tianwenxue-jia* (中国古代天文学者) (*Astronomers in Ancient China*). (One volume of the *Zhongguo tianwenxue-shi daxi* (中国天文学史大系) (*Outline of the History of Chinese Astronomy*). Beijing Zhongguo-kexuejishu-chubanshe. Pp. 226-235 (in Chinese).

Chen Meidong (陈美东), 1992. Yixing. In Du Shiran (杜石然) (ed.). *Zhongguo gudai kexue-jia zhuanji* (中国古代科学家传记) (*Biographies of Scientists in Ancient China*). Volume 1. Beijing, Kexue-chubanshe. Pp. 360-372 (in Chinese).

Chen Meidong, 1998. Yixing. In Jin Qiupeng (金秋鹏) (ed.). *Zhongguo kexue jishu shi, Renwu-juan* (中国科学技术史，人物卷) (*A History of Science and Technology in China, Biographical Volume*). Beijing, Kexue-chubanshe. Pp. 278-294 (in Chinese).

Chen Meidong, 2003. *Zhongguo kexue-jishu-shi, Tianwenxue-juan* (中国科学技术史，天文学卷) (*A History of Science and Technology in China*, *Astronomy Volume*). Beijing, Kexue-chubanshe (in Chinese).

Chen Zungui (陈遵妫), 1980-1989. *Zhongguo tianwenxue-shi* (中国天文学史) (*History of Chinese Astronomy*). Four volumes. Shanghai, Shanghai-renmin-chubanshe (in Chinese).

Kūkai (空海) (= Kōbō-daishi (弘法大師)), 821. *Shingon-fuhō-den* (眞言付法傳)(*Ryaku-fuhō-den* (略付法傳))(*Biographies of the successors of the Mantra sect of Buddhism*) (in classical Chinese). (There are some modern publications, including a critically edited text *Teihon Kōbō-daishi Zenshū* (定本弘法大師全集, Authoritative Complete Works of Kōbō-daishi), Kōyasan, Mikkyōbunka-kenkyūjo, 1991, volume 1, Pp.117-136.)

Li Di (李迪), 1964. *Tang-dai tianwenxue-jia Zhang Sui (Yixing)* (唐代天文学家张遂(一行)) (*Astronomer Zhang Sui (Yixing) of the Tang Dynasty*). Shanghai, Shanghai-renmin-chubanshe (in Chinese).

Liu Xu (劉昫) et al., 945. *Jiu-Tang-shu* (舊唐書)(*Old Version of the Official History of the Tang Dynasty*). (in classical Chinese). (There are some modern publications, including a critically edited text published by Zhonghua-shuju, Beijing, 1975. Its astronomical chapters are included in Zhonghua-shuju (1975-1976).)

Needham, Joseph (with the collaboration of Wang Ling), 1959. *Science and Civilisation in China*. Volume 3, *Mathematics and the Sciences of the Heavens and the Earth*, Cambridge, Cambridge University Press.

Ôhashi Yukio (大橋由紀夫), 1994a. Gokan-ni okeru Indo-tenmongaku denrai-no kanōsei-ni tsuite (後漢におけるインド天文学伝来の可能性について), *Sūgakushi Kenkyū (Journal of History of Mathematics, Japan)*, No.140, 17-32 (in Japanese).

Ôhashi, Yukio, 1994b. Zui-Tō jidai no hokan-hō no sanjutsu-teki kigen (隋唐時代の補間法の算術的起源) (Arithmetical origin of Chinese interpolation of the Sui and Tang Periods). *Kagakusi Kenkyu (Journal of History of Science, Japan)*, Series II, 33 (No.189), 15-24 (in Japanese).

Ôhashi, Yukio, 1995. Daien-reki no hokan-hō ni tsuite (大衍暦の補間法について) (On the interpolation used in the Dayan Calendar). *Kagakusi Kenkyu (Journal of History of Science, Japan)*, Series II, 34(No.195), 170-176 (in Japanese).

Ôhashi, Yukio, 1999a. Historical Significance of Mathematical Astronomy in Later-Han China. In Kim Yung Sik and Francesca Bray (eds.). *Current Perspectives in the History of Science in East Asia*, Seoul, Seoul National University Press, Pp.259-263.

Ôhashi, Yukio, 1999b. Preliminary remarks on the origin of „Mori‟ and „Mieri‟ in Chinese calendars. In *Proceedings of the Fourth International Symposium on the History of Mathematics and Mathematical Education using Chinese Characters*. Maebashi, Maebashi Institute of Technology. Pp.97-102.

Ôhashi, Yukio, 2000. Mori Mieri qiyuan-kao (没日灭日起源考) (On the origin of „Mori" and „Mieri"). *Ziran Kexueshi Yanjiu* (*Studies in the History of Natural Sciences*), 19, 264-270 (in Chinese).

Ôhashi, Yukio, 2007. Formation of Chinese classical mathematical astronomy. *Gaṇita Bhāratī*, 29, 101-115.

Ôhashi, Yukio, 2008. "Chinese mathematical astronomy from ca.4th century to ca.6th century. *Gaṇita Bhāratī*, 30, 27-39.

Ôhashi, Yukio, 2009a. Solar eclipses and Shibukawa Harumi. In Sôma, M., and Tanikawa, K. (eds.). *Proceedings of the Second Symposium on "Historical Records and Modern Science"*. Tokyo, Notional Astronomical Observatory of Japan. Pp.31-44.

Ôhashi, Yukio, 2009b. Yixing de neichafa (一行的内插法)(Interpolation of Yixing). In Wang Fubin (王辅彬) (ed.). *Jiu Tian-ren-zhi-ji Tong Gu-jin-zhi-bian* (究天人之际 通古今之变). Nanking, Guanxi-minzu-chubanshe. Pp. 58-61 (in Chinese).

Osabe Kazuo (長部和雄), 1990. *Ichigyō-zenji no kenkyū* (一行禅師の研究)(*Study of Zen Master Yixing*). Reprint. Tokyo, Hokushindō (originally published in 1963) (in Japanese),

Ouyang Xiu (歐陽修) et. al., 1060. *Xin-Tang-shu* (新唐書)(*New Version of the Official History of the Tang Dynasty*). (in classical Chinese). (There are some modern publications, including a critically edited text published by Zhonghua-shuju, Beijing, 1975. Its astronomical chapters are included in Zhonghua-shuju (1975-1976).) .

Qu Anjing (曲安京), 1997. *Dayan-li* guiying chafen-biao-de chonggou (《大衍历》晷影差分表的重构) (Reconstruction of a difference table of solar shadow in the *Dayan* calendar (AD 724). *Ziran Kexueshi Yanjiu* (*Studies in the History of the Natural Sciences*), 16, 233-244 (in Chinese). (A Japanese version, translated by Yukio Ôhashi, is in *Sūgakushi Kenkyū* (*Journal of the History of Mathematics, Japan*), No.153, 18-29 (1997).)

Qu Anjing, 2005. *Zhongguo lifa yu shuxue* (中国历法与数学)(*The Chinese Calendar and Mathematics*). Beijing, Kexue-chubanshe (in Chinese).

Qu Anjing, Ji Zhigang, and Wang Rongbin (曲安京、纪志刚、王荣彬), 1994. *Zhongguo gudai shuli tianwenxue tanxi* (中国数理古代天文学探析)(Researches on Mathematical Astronomy in Ancient China). Xi"an, Xibei-daxue-chubanshe (in Chinese).

Ruan Yuan (阮元), 1799. *Chouren-zhuan* (疇人傳)(*Biographies of Astronomers*) Volumes 14-16 (republished by Shijie-shuju (世界書局), Taipei, 1962) (in classical Chinese).

Yabuuti Kiyosi (= Yabuuchi Kiyoshi (藪内清), 1990. *Zōho-kaitei Chūgoku no tenmon rekihō* (中国の天文暦法)(Enlarged *and Revised Edition of the History of Astronomical Calendars in China*). Tokyo, Heibonsha (in Japanese).

Zanning (贊寧), 988. *Song gaoseng-zhuan* (宋高僧傳)(*Biographies of Eminent Buddhist Monks Compiled in the Song Dynasty*). Volume 5 (in classical Chinese). (There is a critical edition published by Zhonghua-shuju, Beijing, 1987. It is also included in the *Taishō-shinshū Daizōkyō* (大正新脩大蔵経, *The Tripitaka in Chinese*), Volime 50, The Taisho Issai-kyo Kanko Kwai (大正一切經刊行會), Tokyo, 1927.)

Zhonghua-shuju (中華書局) (eds.), 1975-1976. *Lidai Tianwen-lüli-deng-zhi Huibian* (歷代天文律暦等志彙編)(*Collection of the Chapters on Astronomy, Acoustics and the Calendar in the Official Dynastic Histories)*. Ten volumes. Beijing, Zhonghua-shuju (in classical Chinese).